Two envelopes problem

(exchange paradox)

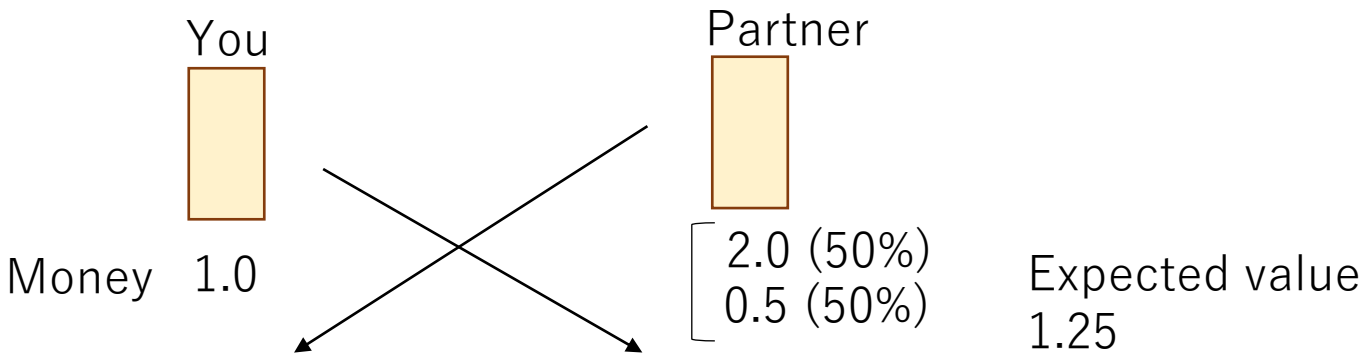
Psychology of misunderstand

Complete solution

Two envelopes problem(exchange paradox)

Question

There are two envelopes containing money, and one is distributed to you.
One envelope contains twice the amount of the other.
Is it better to exchange them?



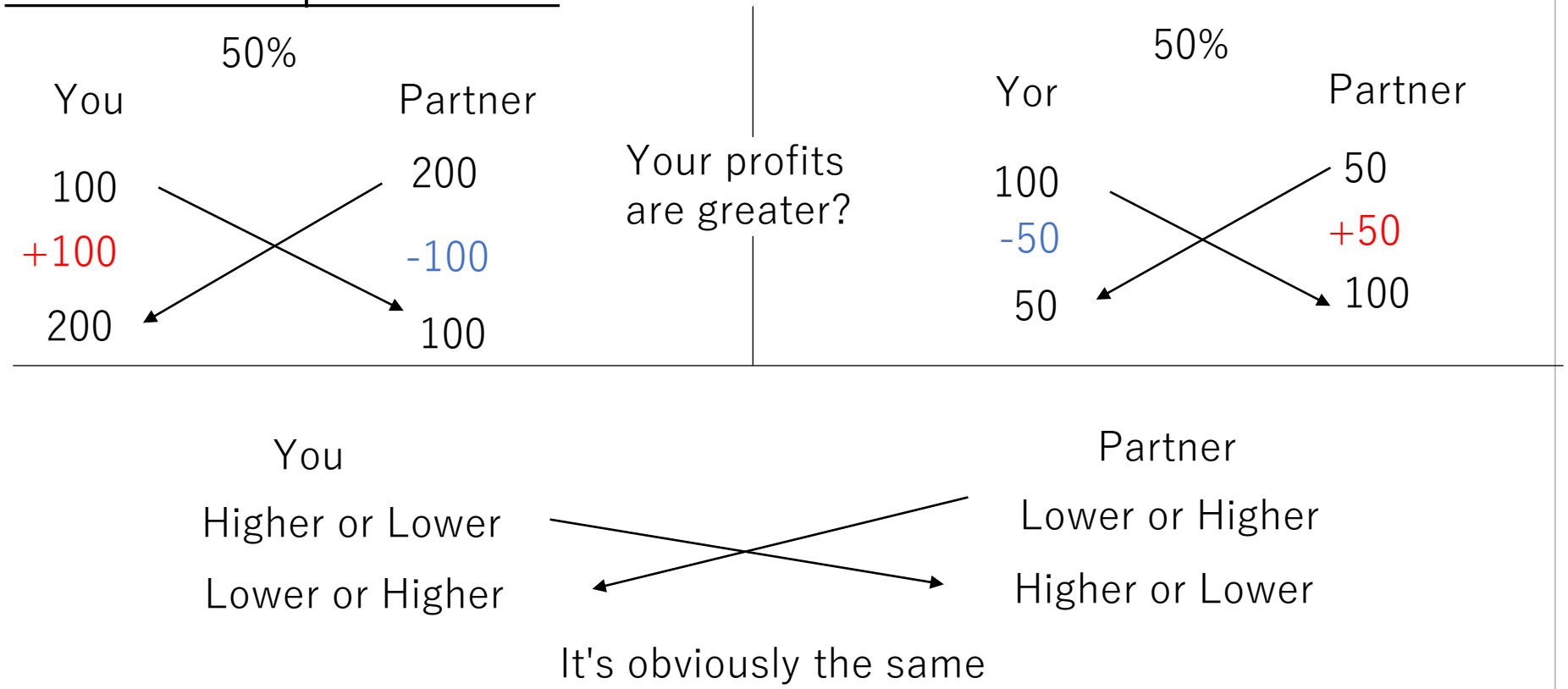
Exchange partner is under the same conditions.

It is clearly strange that each party would gain an expected value of 1.25 times.

A great detective completely solves the two-envelope problem (exchange paradox).
Not only does it explain what is correct, but it also explains the psychology of why it is wrong.
There are two envelopes containing money, and one is distributed to you.
One envelope contains twice the amount of the other.
Is it better to exchange them?
There is a 50% chance that you will get double the amount, and a 50% chance that you will get 0.5 times the amount.
We can calculate that the expected value is 1.25 times.
However, the exchange partner is under the same conditions.
It is clearly strange that each party would gain an expected value of 1.25 times.

Two envelopes problem(exchange paradox)

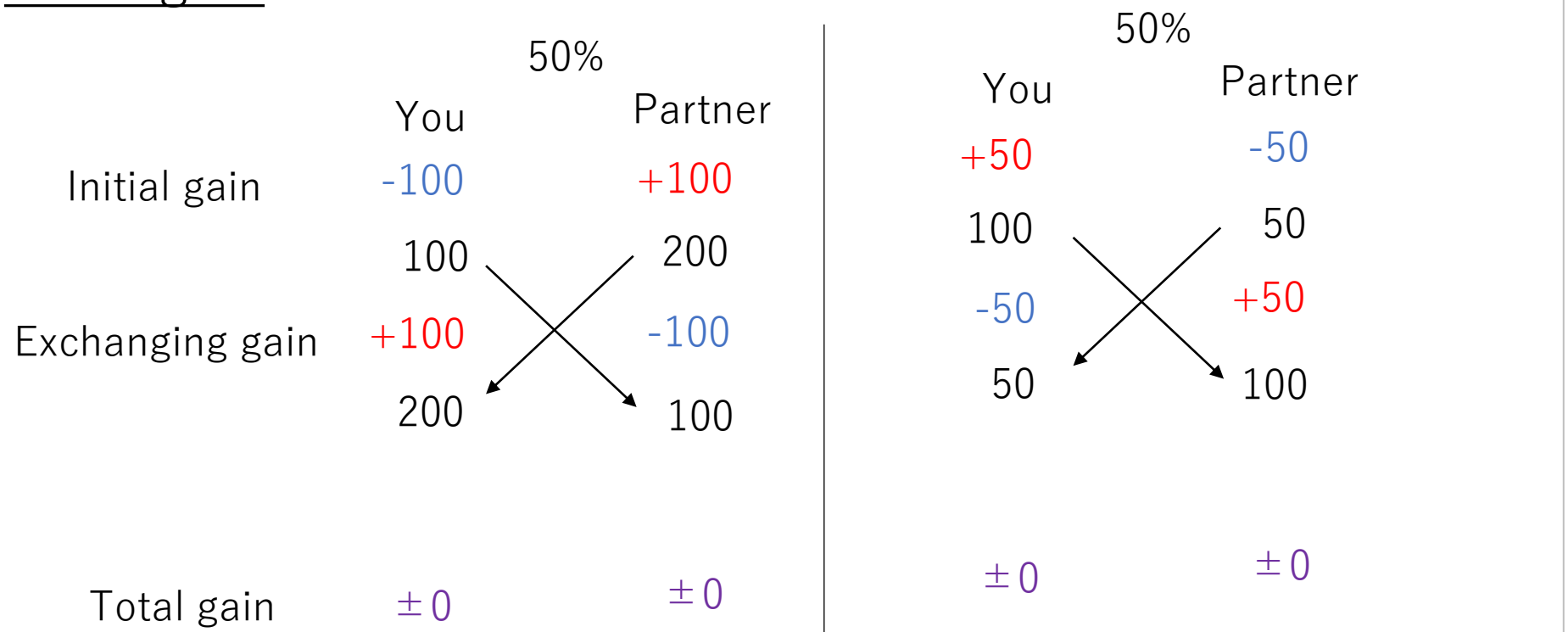
Common explanations



Let's say you were given 100 yen.
If the other person has 200 yen, when you exchange, you will gain +100 yen and the other person will lose -100 yen.
If the other person has 50 yen, when you exchange, you will lose -50 yen and the other person will gain +50 yen.
The expected value is +25 yen for you and -25 yen for the other person.
As expected, it seems like you would gain more if you exchanged.
However, whether you exchange or not, it will only be either a higher or lower amount.
It is clear that the amount you end up with is the same for both people.

Two envelopes problem(exchange paradox)

Initial gain



An important oversight is the initial gain or loss.
If you have 100 yen and the other person has 200 yen, you will gain +100 yen after exchanging.
However, initially you were -100 yen less than the other person.
If you add the initial -100 yen and the +100 yen from the exchange, it comes to ±0 yen.
If you have 100 yen and the other person has 50 yen, you will lose -50 yen after exchanging.
However, initially you were +50 yen more than the other person.
If you add the initial +50 yen and the -50 yen from the exchange, it comes to ±0 yen.
Even if you don't know which case it is, the exchange will still be ±0 yen.

Two envelopes problem(exchange paradox)

Independence of Probabilities

Final profit = initial profit + profit before and after the exchange

What this question is really asking is "final profit“,
not "profit before and after the exchange".

There is no doubt that "profit before and after the exchange"
have an expected value of 1.25 times each other.

If you make a profit from an exchange, you are losing money initially.

The probability of “Initial profit" and the probability of “Exchange profit”
are not independent.

Because we think the two probabilities are independent,
we think it is better to exchange.

"Final profit= initial profit/loss + profit before and after the exchange".
What this question is really asking is "final profit", not "profit before and after the exchange".
There is no doubt that "profit before and after the exchange" have an expected value of 1.25 times each other.
However, if you make a profit from an exchange, you are losing money initially.
The probability of “Initial profit" and the probability of “Exchange profit” are not independent.
Because we think the two probabilities are independent, we think it is better to exchange.
When calculating probability, we tend to assume that they are independent for the time being.
This is because independent probabilities are easier to calculate.

Two envelopes problem(exchange paradox)

Only look at percentages

Price 100,000 → 90,000

1,000,000 → 100,000

Is this a good deal?

Some people only look at the percentage increase or decrease and don't really look at the absolute values.

The fact that is worth 1 million yen is just the store manager's opinion, right?
Is there really data that proves that one cat has a market value of 1 million yen?

All cats are cute, so it's wrong to rank them.

Additionally, some people only look at the percentage increase or decrease and don't really look at the absolute values.
At a pet shop, a cat that cost 100,000 yen was 10% off, now costing 90,000 yen.
Or, a luxury cat that cost 1 million yen was 90% off, now costing 100,000 yen.
Many people would feel that 90% off is a better deal.
But the fact that the cat is worth 1 million yen is just the store manager's opinion, right?
Is there really data that proves that one cat has a market value of 1 million yen?
All cats are cute, so it's wrong to rank them.

Two envelopes problem(exchange paradox)

Only percentages are displayed

Sometimes only the percentage of increase or decrease is shown.

99.9%
Sterilization

If you keep the initial number of bacteria high,
you can easily eliminate 99.9% of bacteria.

Ordinary toilet paper

Tap water from a shower toilet

Bare hands

} Eliminates 99.9%
of E. coli in the anus

↑ Your hands have the special ability to disinfect.

This is the paradox.

For one thing, sometimes only the percentage of increase or decrease is shown.
I'm sure you've seen the term "99.9% bacteria elimination“.
However, I've never seen any indication of how many bacteria have been reduced to.
If you keep the initial number of bacteria high, you can easily eliminate 99.9% of bacteria.
Ordinary toilet paper can eliminate 99.9% of E. coli in the anus.
Tap water from a shower toilet also has the effect of eliminating 99.9% of bacteria.
If you wipe your buttocks with your bare hands, your hands have the special ability to eliminate bacteria.
This is the paradox.
That's all.

Contact Information

For inquiries,

please contact us here.

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