

Sleeping Beauty problem

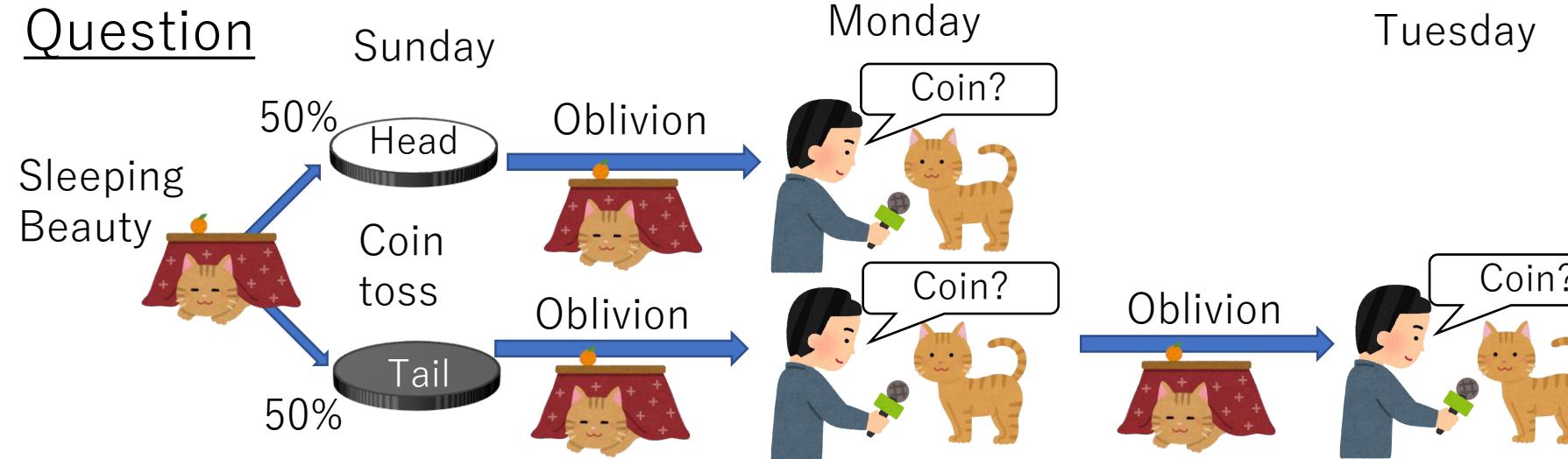
Psychology of misunderstand

Complete solution



Sleeping Beauty problem

Question



On Sunday, a coin is tossed while Sleeping Beauty is asleep.

If the coin lands on heads, she will be woken up on Monday and questioned.

If the coin lands on tails, she will be questioned on Monday and Tuesday.

She has been given a memory eraser and does not know what day of the week it is.

What would her say is the probability that the coin would land on heads?

A great detective solves the Sleeping Beauty problem once and for all.

We'll not only explain what is right, but also the psychology of why we make mistakes.

A fair coin is tossed once on Sunday while Sleeping Beauty is asleep.

If the coin lands on heads, Sleeping Beauty will be woken up on Monday and questioned.

If the coin lands on tails, Sleeping Beauty will be woken up on Monday and Tuesday and questioned.

Every day, Sleeping Beauty is given a memory erasing drug and is not told what day of the week it is.

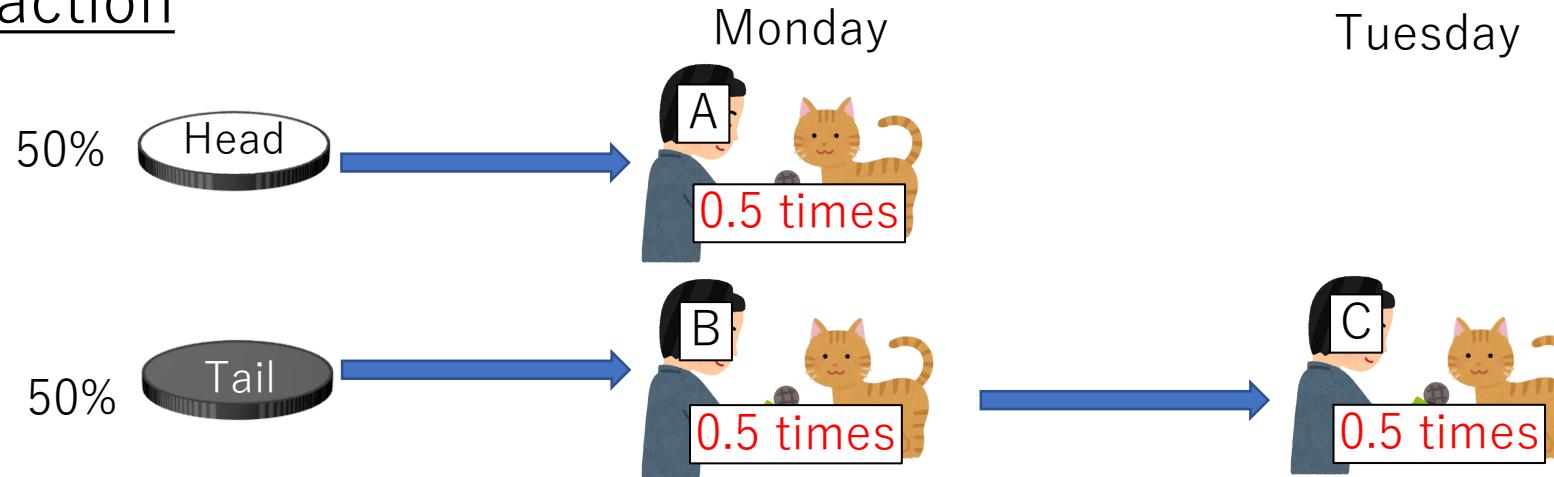
Imagine you are Sleeping Beauty who has just woken up.

The rules of the experiment have been explained to you, but you don't know what day of the week it is.

What do you think the probability of the coin landing on heads is?

Sleeping Beauty problem

1/3 Fraction



Frequency $A : B : C = 0.5 \text{ times} : 0.5 \text{ times} : 0.5 \text{ times}$
 $= 1/3 : 1/3 : 1/3$
 Head Tail Tail

$\text{Head} = 1/3$

It seems that some people say the answer is 1/3, while others say it's 1/2.

Let's explain how we calculate 1/3.

When Sleeping Beauty wakes up, she understands that she is in one of three situations.

A (heads, Monday), or B (tails, Monday), or C (tails, Tuesday).

Calculate the frequency of the three situations.

A occurs 0.5 times.

The remaining 0.5 times, both B and C occur.

All three situations occur with the same frequency of 0.5.

Since the probability of each is 1/3, the probability of the coin landing on heads is 1/3.

Sleeping Beauty problem

1/2 Fraction

(1) They only partially understood the experiment.

I understand that because it is a fair coin, the probability of heads is 50%.

(2) They misunderstood what was being asked.

"What is the probability of the coin landing on heads is?"

The Questioner : I wanted to hear the results of the coin tossed on Sunday.

Sleeping beauty : I interpreted this as me being asked about
the probability of getting heads when flipping a coin.

Head = 1/2

On the other hand, there are two possible answers for 1/2.

(1) They only partially understood the experiment.
(2) They misunderstood what was being asked.

The first reason is that, because the coin was fair, they only thought about 50%.

The second reason is that they misunderstood the question, "What is the probability that the coin will land on heads?"

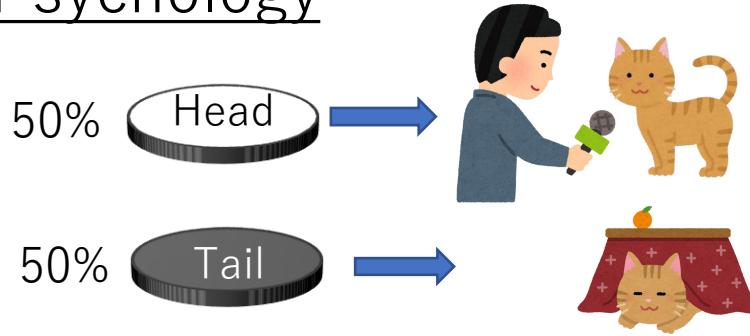
The questioner may have wanted to know the result of the coin tossed on Sunday.

Let's say Sleeping Beauty interpreted the question as meaning that they were being asked about the probability of heads landing on a coin toss.

If that's the case, then 1/2 is the correct answer.

Sleeping Beauty problem

Psychology



If the coin lands on heads, it is woken up,
and if it lands on tails, it is not woken up.

The questioner : " Is the coin heads or tails?"

Sleeping beauty : "Head"

The questioner : "What is the probability of the coin landing on heads is?"

Sleeping beauty : "50%"

I don't think you should use the word "probability" to ask about a definite result.

I interpret this as asking about the probability of heads appearing when coin toss.

The psychology of misunderstanding is easier to understand with the following example.

Let's say Sleeping Beauty will be woken up if the coin lands on heads, and will not be woken up if it lands on tails.

If she were asked, "Is the coin heads or tails?" she would probably answer "heads".

Because once she has been woken up, there is no possibility of it landing on tails.

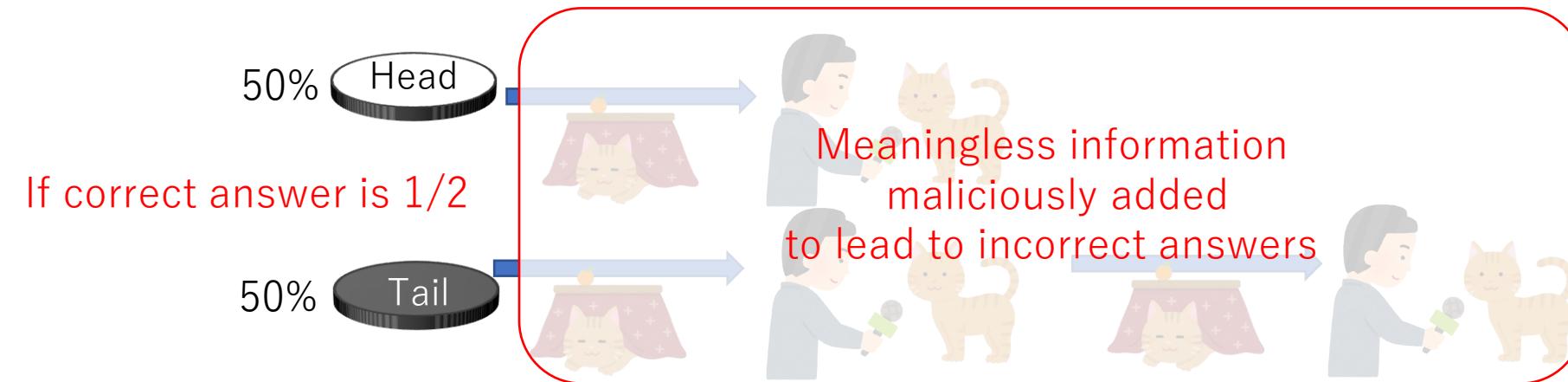
However, if she were asked, "What is the probability of the coin landing on heads?" she would probably answer 50%.

I don't think you would use the word "probability" to ask about a definite outcome.

I would interpret this as her asking about the probability of heads appearing when a coin is tossed.

Sleeping Beauty problem

Theory of Good Human Nature



The questioner is a good person: It can't be $1/2$, so the correct answer is $1/3$

The questioner is a bad guy: This is a trick question, so the correct answer is $1/2$

The $1/3$ and $1/2$ camps can also be explained by the theory of human goodness and evil.

If the correct answer to the question is $1/2$, then all of the complicated exchanges were meaningless information.

It was information added with malicious intent to lead you to the incorrect answer.

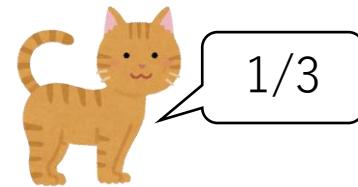
If you believe the person asking the question was a good person, then $1/2$ is not possible, so $1/3$ is the correct answer.

If you believe the person asking the question was a bad person, then it was a trick question and $1/2$ is the correct answer.

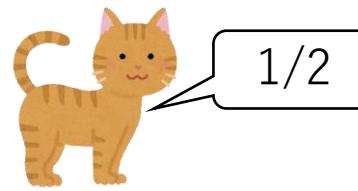
However, even those who say $1/2$ is the correct answer are underestimating human malice.

Sleeping Beauty problem

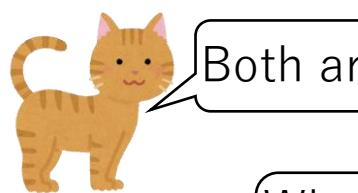
Best practice



The correct answer was $1/2$.
How do you feel, having been fooled and
having to make a waste of calculations?



The correct answer was $1/3$.
How do you feel, not being able to
calculate such a simple probability?



Both are correct depending on your interpretation



What are you going to do
if you get so serious about a question like this?



If you answer "1/3," you will hear the following response.

The correct answer was $1/2$. How do you feel, having been fooled and having to make a waste of calculations?

If you answer "1/2," you will hear the following response.

The correct answer was $1/3$. How do you feel, not being able to calculate such a simple probability?

If you refuse to answer, you will be told the same thing.

If you carefully explain that both answers are correct, depending on how you interpret them, you will hear the following response.

What are you going to do if you get so serious about a question like this?

In other words, the right thing to do is to force your opponent to shut up.

That's all.

Contact Information

For inquiries,
please contact us here.

<https://ultagi.org/>