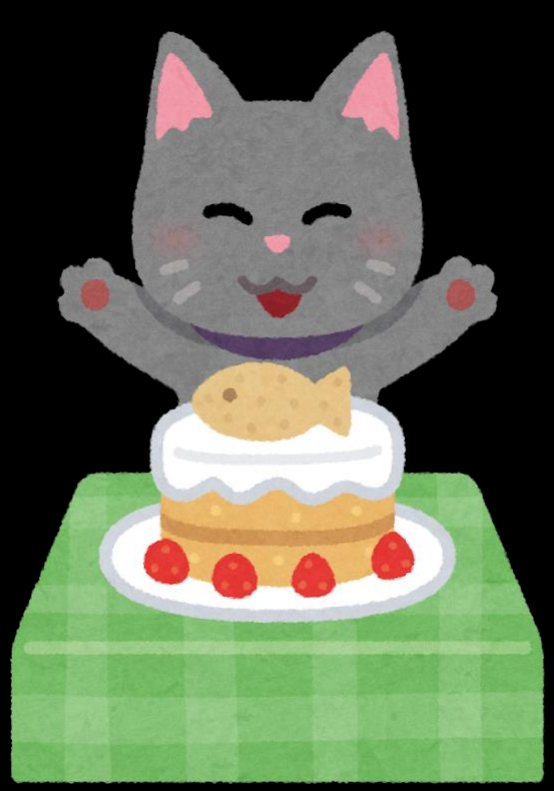
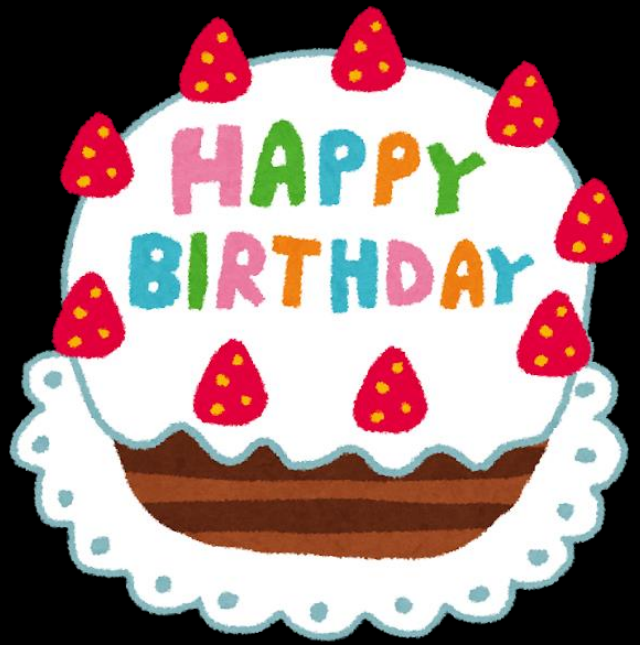
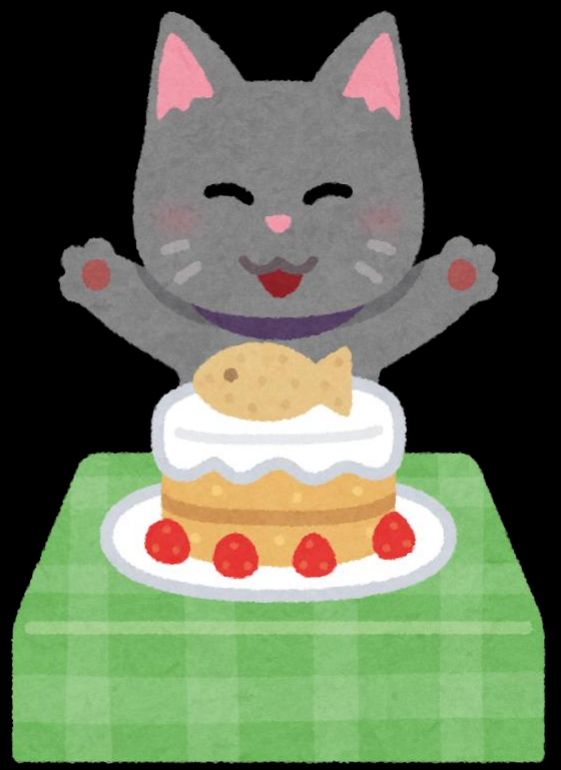
Birthday Paradox

Psychology of misunderstand

Complete solution







Birthday Paradox

Question

What is the probability that two people in your class have the same birthday?

A year consists of 365 days.

Correct answer

If there are 23 people in a class,
there is a 51% chance that two people will have the same birthday.

If there are 30 people in a class,
there is a 71% chance that two people will have the same birthday.

The birthday paradox is that it feels like the probability is higher than you think.

I will explain the psychology behind this feeling.

A great detective completely solves the birthday paradox.
By the way, it also solves the paradox of the 66-type random gacha.
The question asks what the probability is that two people in a class have the same birthday.
Assume there are 365 days in a year.
If there are 23 people in a class, then the probability that two people have the same birthday is 51%.
If there are 30 people in a class, then the probability that two people have the same birthday is 71%.
The birthday paradox is when you feel that the probability is higher than you thought.
We will explain the psychology behind why you feel this way.

Birthday Paradox

Process

(1) Interpretation

Problems expressed in words are open to interpretation.

In the birthday question, it is likely that there is an error in "interpretation“.

(2) Calculation

If the problem is expressed as a formula,
only calculations are required and fewer people will make mistakes.

We will think of this as two separate processes: "interpretation" and "calculation“.
This is because a problem expressed in words leaves room for interpretation.
If the problem were expressed as a mathematical formula, fewer people would make mistakes since all they would have to do was calculate.
In the birthday problem, we can predict that an error is occurring in "interpretation“.

Birthday Paradox

Characteristics of the interpretation process

Question

What is the total price for 16 items that cost 25 yen each?

Interpretation (1)

$$4 \times 4 \times 25 = 4 \times 100 = 400$$

Interpretation(2)

$$\begin{array}{r} 16 \\ \times 25 \\ \hline 80 \\ 32 \\ \hline 400 \end{array}$$

There are so many numbers that need to be temporarily memorized, it's difficult.

Before performing calculations, humans try to interpret a problem in a way that makes it as easy to compute as possible.

There are distinctive features in the process that humans use to "interpret" problems.
For example, what is the total price of 16 items that cost 25 yen each?
I think most people would interpret this as 4 x 4 x 25 = 4 x 100 = 400 and calculate it.
I don't think there are many people who would try to calculate 16 x 25 in their heads in the same way as doing it by hand.
This is because there are so many numbers that have to be temporarily remembered, which makes it difficult.
Before calculating, humans try to interpret the problem in a way that makes it as easy as possible to calculate.

Birthday Paradox

The process of interpreting the birthday problem

Question: If there are 365 days in a year, what is the probability that two people in a class of 30 have the same birthday?

Awareness
- The more days there are, the harder it is to match birthday.
  If it's only 1 day, it's 100%.
- The more people there are, the easier it is to match birthday.
  If it's only 0 people, it's 0%.

30people / 365day = 8%

An interpretation that is as easy to calculate as possible using two numbers.

Even though I realize how many combinations there are, I can't do the math in my head right away, so the only number I have in my head is 8%.

When I hear that the correct answer is 81%, it feels bigger than I expected.

In the birthday question, only two numbers are visible: 365 days and 30 people.
Using these two numbers, try to interpret it in a way that is as easy as possible to calculate.
You realize that the more days there are in a year, the less likely it is that the birthdays will match, and if it's 1 day, the probability is 100%.
You also realize that the more people there are, the easier it is for the birthdays to match, and if there are zero people, the probability is 0%.
If you try to express this relationship using simple arithmetic operations, you get 30/365=approximately 8%.
Some people may realize that 8% is not accurate because combination calculations are required.
However, if you cannot do the calculation in your head right away, the only number in your head is 8%.
If you are told the correct answer, 81%, you will feel that it is bigger than you expected.

Birthday Paradox

Gacha Paradox

Question: How many times does it take to complete the gacha that randomly gives out 10 different items?

Awareness
- It takes at least 10 times.
- The more types there are, the more times they will be needed.

Simple interpretation: 10 times

Expected value: 29 times

It feels like more draws are needed than you thought.

A similar paradox occurs with gacha probabilities.
How many times do you need to draw to complete a gacha that randomly gives you 10 types?
You need to draw at least 10 times.
All you can see is the number 10 types, and you realize that the larger it is, the more times you'll need to draw.
A simple interpretation is that you can only predict 10 draws.
The expected value is 29 draws.
It feels like more draws are needed than you thought.

Birthday Paradox

Gacha Paradox

People who think it's mostly as expected

(1) They have done the calculations before.

(2) They have actually experienced that they can complete the set by drawing about 3 times the number of types.

I have the number "3" in my brain.

The estimated number of times required is 10 types x 3 = 30 times.

This is a dangerous way of thinking.

However, there are some people who feel that this is roughly as expected.
These people are in one of two places.
(1) They have done the calculations before.
(2) They have actually experienced that they can complete the set by drawing about three times the number of types.
The second person has the number "3" in their head.
They estimate that 10 types x 3 = 30 times is the number of times they need to draw.
This is a dangerous way of thinking.

Birthday Paradox

Gacha Paradox

Question: How many times does it take to complete the gacha that randomly gives out 66 different items?

Empirical Estimation :

66 types x 3 = 198 times

Expected value: 315 times (Approximately 5 times the 66 types)

You might misestimate the number of draws you'll need and end up in a bad situation.

But it's also a great sales technique.

This is the tool of evil civilization.

There are 66 different random gacha in the world.
Empirically, we can estimate that you'll need to draw 66 x 3 = 198 times.
The actual expected value is 315 draws, which is about five times the 66 types.
You might misestimate the number of draws you'll need and end up in a bad situation.
But it's also a great sales technique.
This is the tool of evil civilization.
That's all.

Contact Information

For inquiries,

please contact us here.

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